OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-12 : CO-ORDINATE GEOMETRY OF THREE DIMENSIONS

UNIT TEST-1

1. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2},1,\lambda\right)$ and $(-2, 0, 1)$ are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1}=\frac{y-1}{2}=\frac{z+7}{2}$ is _____.

2. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

3. Let the image of the point $P(1, 2, 3)$in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are $(6, 10, 7)$, then the square of the area of the triangle PQR is _____.

Hints and Solutions

1. (09.00)

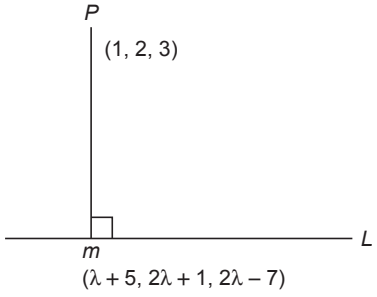
$$\left|\frac{-4+0-6+7}{7}\right|=\left|\frac{15-6\lambda}{7}\right|$$

$$\frac{3}{7}=\left|\frac{15-6\lambda}{7}\right|$$

$$\lambda = 2 \text{ or } 3$$

$$\lambda_1 = 3, \lambda_2 = 2$$

$$(\lambda_1 - \lambda_2, \lambda_2, \lambda_1) = (1, 2, 3)$$



$$\overrightarrow{PM}\cdot(\hat{i}+2\hat{j}+2k)=0$$

$$(\lambda + 4) + 2(2\lambda - 1) + 2(2\lambda - 10) = 0$$

$$\Rightarrow \quad 9\lambda = 18 \text{ or } \lambda = 2$$

$$\text{Distance} = \sqrt{6^2+3^2+6^2} = 9$$

2. (11.00)

$$\begin{vmatrix} x-2 & y+1 & z \\ 0 & 1 & -1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$$3(x - 2) + 3(-y - 1 - z) = 0$$

$$x - 2 - y - 1 - z = 0$$

$$x - y - z = 3$$

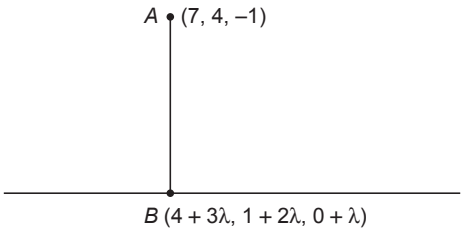
Direction of line

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -7 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(1-7) - \hat{j}(-3+7) + \hat{k}(-3+1)$$

$$= -6\hat{i} - 4\hat{j} - 2\hat{k}$$

(4, 1, 0) is a point on line of intersection



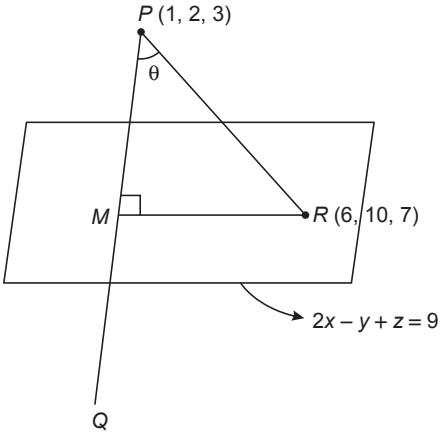
$3(3\lambda - 3) + 2(2\lambda - 3) + 1(\lambda + 1) = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\therefore \quad B = (7, 3, 1)$

$\therefore \quad \alpha + \beta + \gamma = 7 + 3 + 1 = 11$

39. (594.00)



R lies on plane

$$PR = \sqrt{5^2 + 8^2 + 4^2} = \sqrt{105}$$

$$\cos\theta = \frac{(5\hat{i} + 8\hat{j} + 4\hat{k})(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{105}\sqrt{6}}$$

$$= \frac{6}{\sqrt{630}}$$

$$\text{Area } (\Delta PQR) = 2\,\text{area}(\Delta PMR)$$

$$= 2 \cdot \frac{1}{2}(PR)^2 \sin\theta\cos\theta$$

$$= 105 \cdot \frac{6}{\sqrt{630}} \cdot \frac{\sqrt{594}}{\sqrt{630}}$$

$$= \sqrt{594}$$